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Dynamic Pricing with Procedural and Substantive Fairness

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(Joint work with *Dan Qiao* and *Yu-Xiang Wang*)

Outline

- Motivation
  - Two fairness concerns
  - Doubly-fair and profitable policies
- Problem Setup
  - Online pricing with two fairness concerns.
- Algorithm
  - A policy-elimination scheme
  - Regret and Unfairness bounds

References:
Xu, Jianyu, Dan Qiao, and Yu-Xiang Wang. "Doubly Fair Dynamic Pricing." *AISTATS* 2023.

Two Unfairnesses while Booking/Boarding Flight

- While *booking* a flight (on a 3rd –party website)…
  - Your colleague C gets a cheaper offer
  - **Proposed** prices are not equal
  - A **procedural unfairness**

- While *boarding* a flight …
  - Your neighbor N paid at a cheaper price
  - **Accepted** prices are not equal
  - A **substantive unfairness**

Fixed-Price Policy: A straightforward solution

- Two fairness concerns:
  - Procedural unfairness: $U(p_A, p_B) := |\mathbb{E}[p_A] - \mathbb{E}[p_B]|$
  - Substantive unfairness: $S(p_A, p_B) := |\mathbb{E}[p_A \,|A\ accept\ p_A] - \mathbb{E}[p_B|B\ accept\ p_B]|$

- $p_A = p_B$ eliminate both unfairnesses.
  - Optimal price: $p^* = \arg\max_p p \cdot \left(D_A(p) * Q(A) + D_B(p) * Q(B)\right)$
  - $Q(A)$ and $Q(B)$ are the portion of Group A and B.

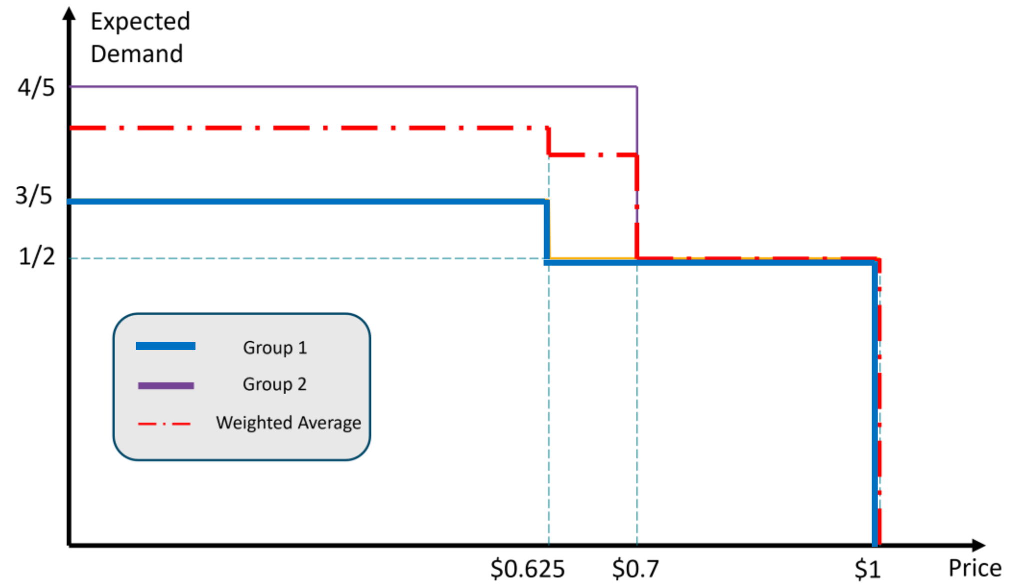
- But can we do better?
  - … if p_A and p_B are generally random.
  - Note: same distributions of p_A and p_B do not work for substantive fairness.

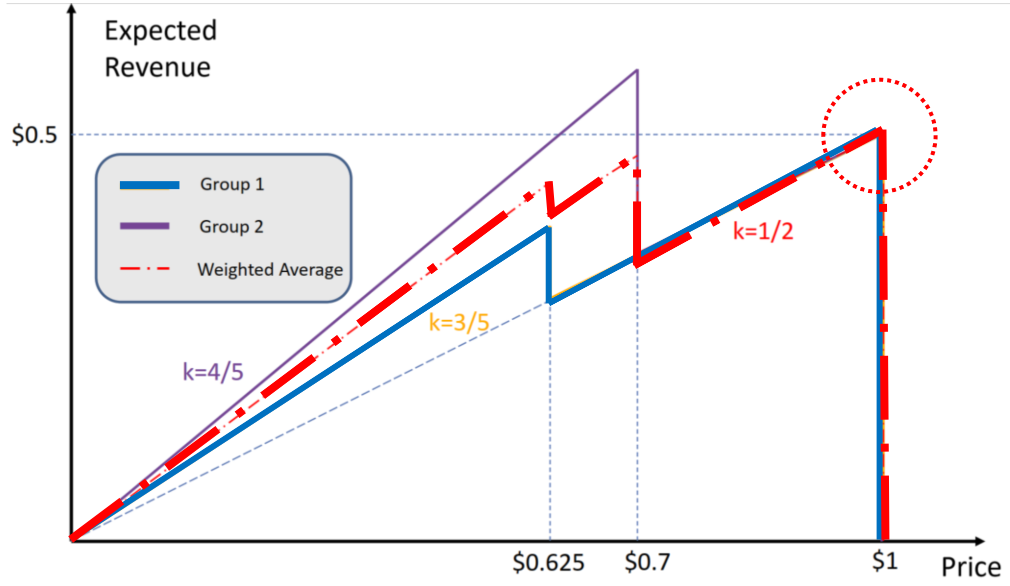
Example: Randomized Prices are More Profitable

- Consider the following example:

Acceptance Rate	\$0.625	\$0.7	\$1
G_1 (30%)	3/5	1/2	1/2
G_2 (70%)	4/5	4/5	1/2

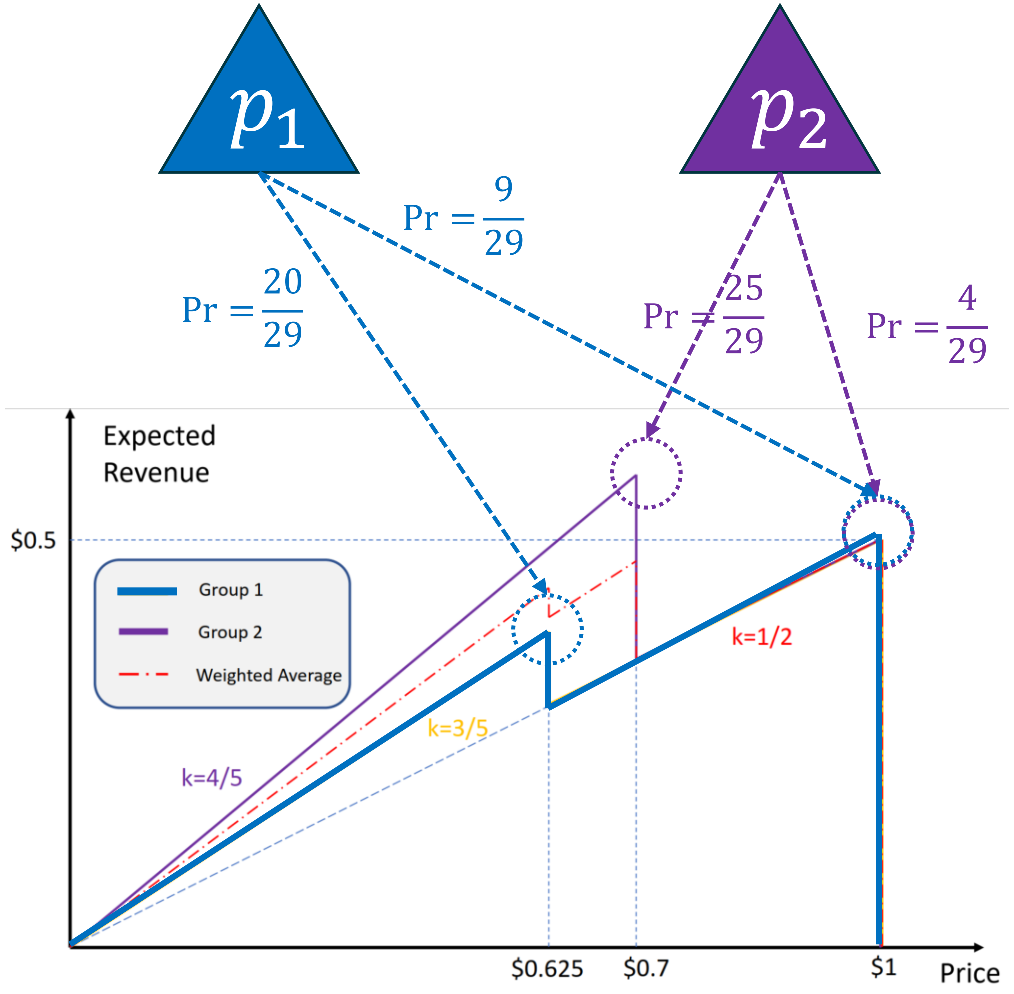
- Let $p_1 = p_2$ to meet the fairnesses
  - $p^* = \arg\max_p D_1(p) * 0.3 + D_2(p) * 0.7 = \$1$
- If $p_1 = p_2 = \$1$, then
  - $D_1(p_1) = D_2(p_2) = 0.5$
  - Revenue = \$0.5
- However, if $p_1 \sim \mathbb{P}_1, p_2 \sim \mathbb{P}_2$ are **random**, then …





Example: Randomized Prices are More Profitable

- Let $p_1 = \begin{cases} \$\,0.625 & \left(\Pr = \frac{20}{29}\right) \\ \$\,1 & \left(\Pr = \frac{9}{29}\right) \end{cases}$ and $p_2 = \begin{cases} \$\,0.7 & \left(\Pr = \frac{25}{29}\right) \\ \$\,1 & \left(\Pr = \frac{4}{29}\right) \end{cases}$
- Procedural fairness holds as $\mathbb{E}[p_1] = \mathbb{E}[p_2] = \frac{43}{58}$.
- Substantive fairness holds as $\mathbb{E}[p_1|buy] = \mathbb{E}[p_2|buy] = \frac{8}{11}$
- And **profit increases** !!
  - $\mathbb{E}\big[p_1 \cdot 1[p_1\ accepted]\big] \cdot 0.3 + \mathbb{E}\big[p_2 \cdot 1[p_2\ accepted]\big] \cdot 0.7$
  - $= \$\frac{74}{145} \approx \$0.5103 > \$\,0.5.$



Seek for Optimal Price Distribution

- Find optimal price → optimal *distribution* of prices.
- Question 1: What is the best fair distribution?

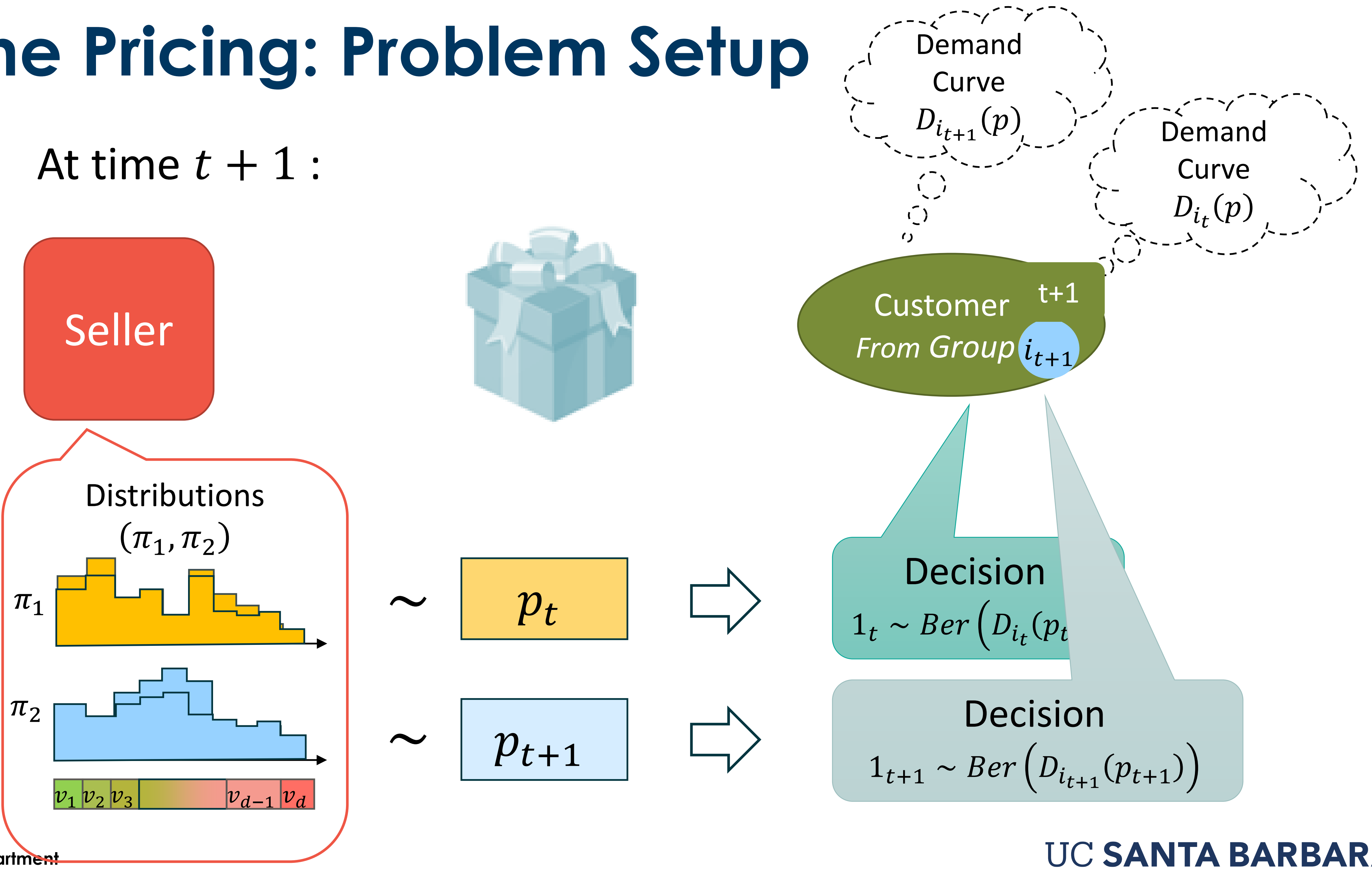
$$\max_{\pi_1,\pi_2} R(\pi_1,\pi_2) \coloneqq \mathbb{E}_{p_1\sim\pi_1}[p_1 \cdot D_1(p_1)] \cdot Q_1 + \mathbb{E}_{p_2\sim\pi_2}[p_2 \cdot D_2(p_2)] \cdot Q_2$$
$$s.t. U(\pi_1,\pi_2) = 0, \qquad S(\pi_1,\pi_2) = 0$$

Here $D_i(p)$ are the demand function of each group i.

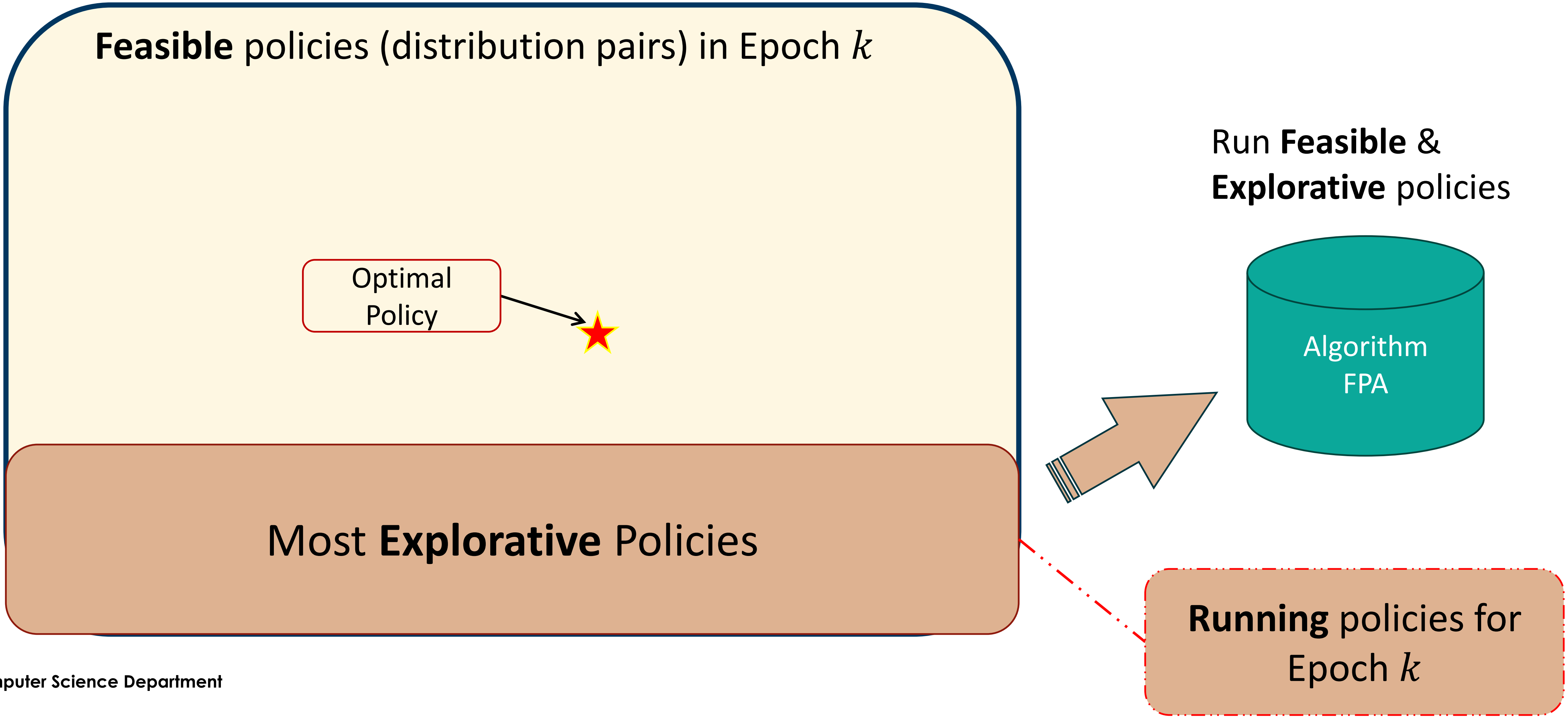
- Question 2: How to **learn** them over time?
  - Unknown $D_1(p)$ and $D_2(p)$ → Unknown $R(\pi_1,\pi_2)$ and $S(\pi_1,\pi_2)$
  - But we can learn from customers' buy/not buy decisions.
  - An online dynamic pricing problem.

Online Pricing: Problem Setup

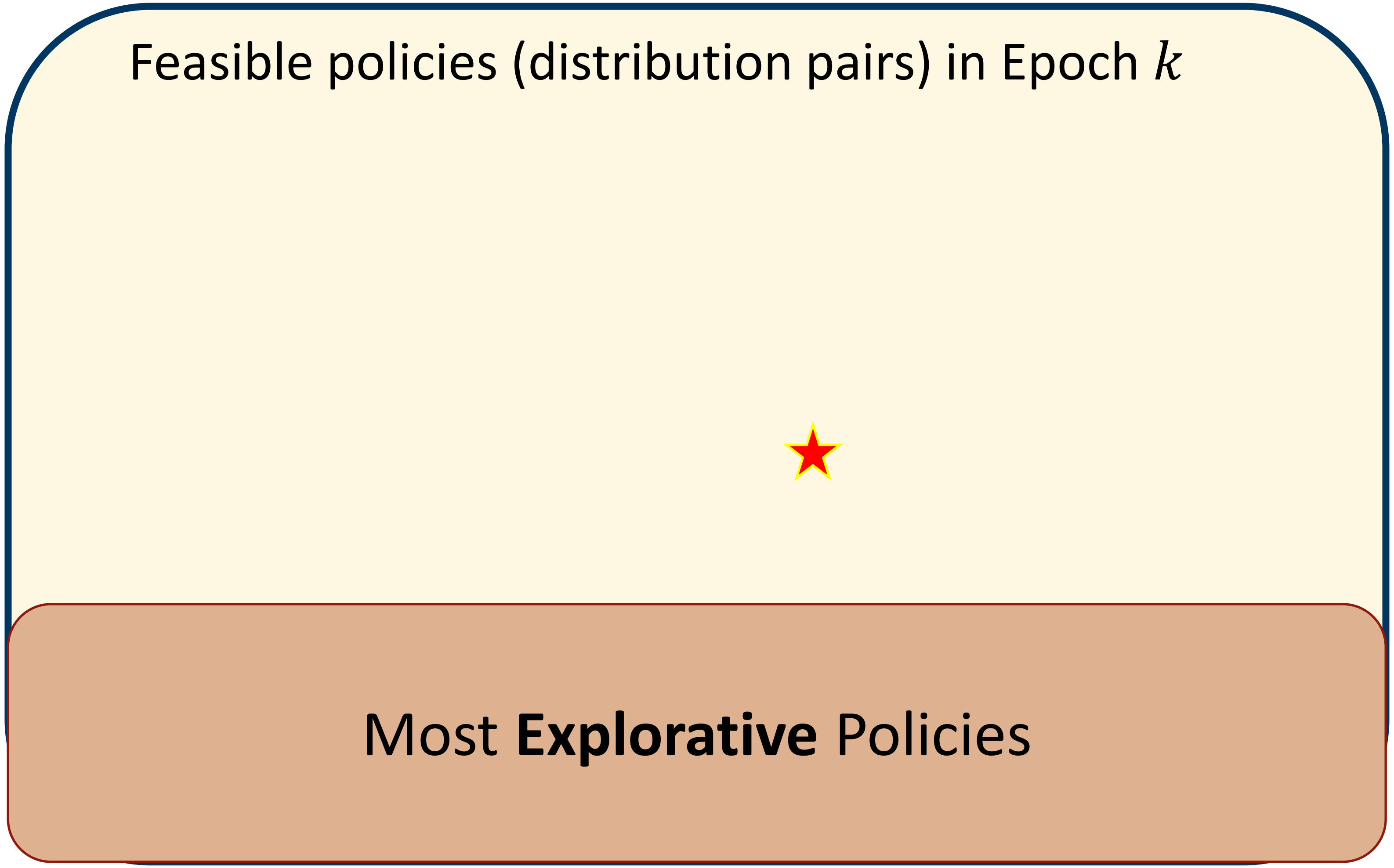
At time $t+1$:



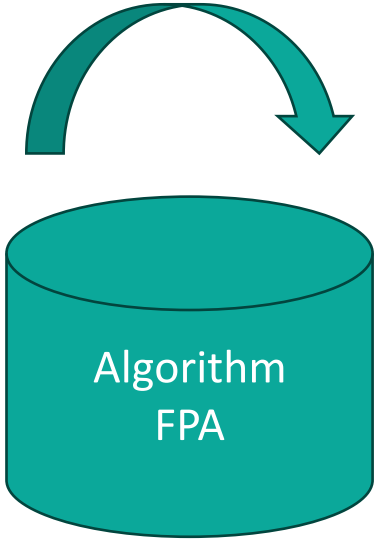
Algorithm: Epoch-based Policy Elimination



Algorithm: Epoch-based Policy Elimination



Gather feedback and solve constraint optimization problems



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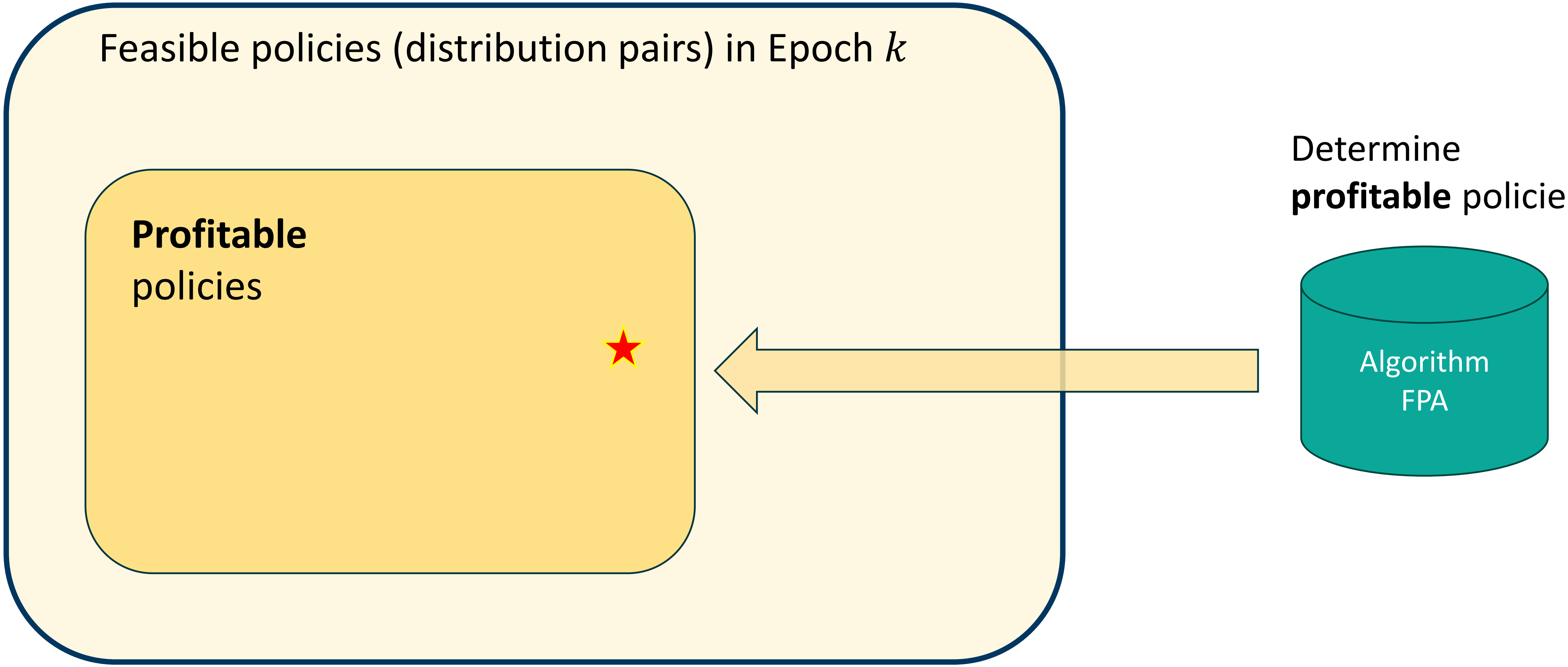

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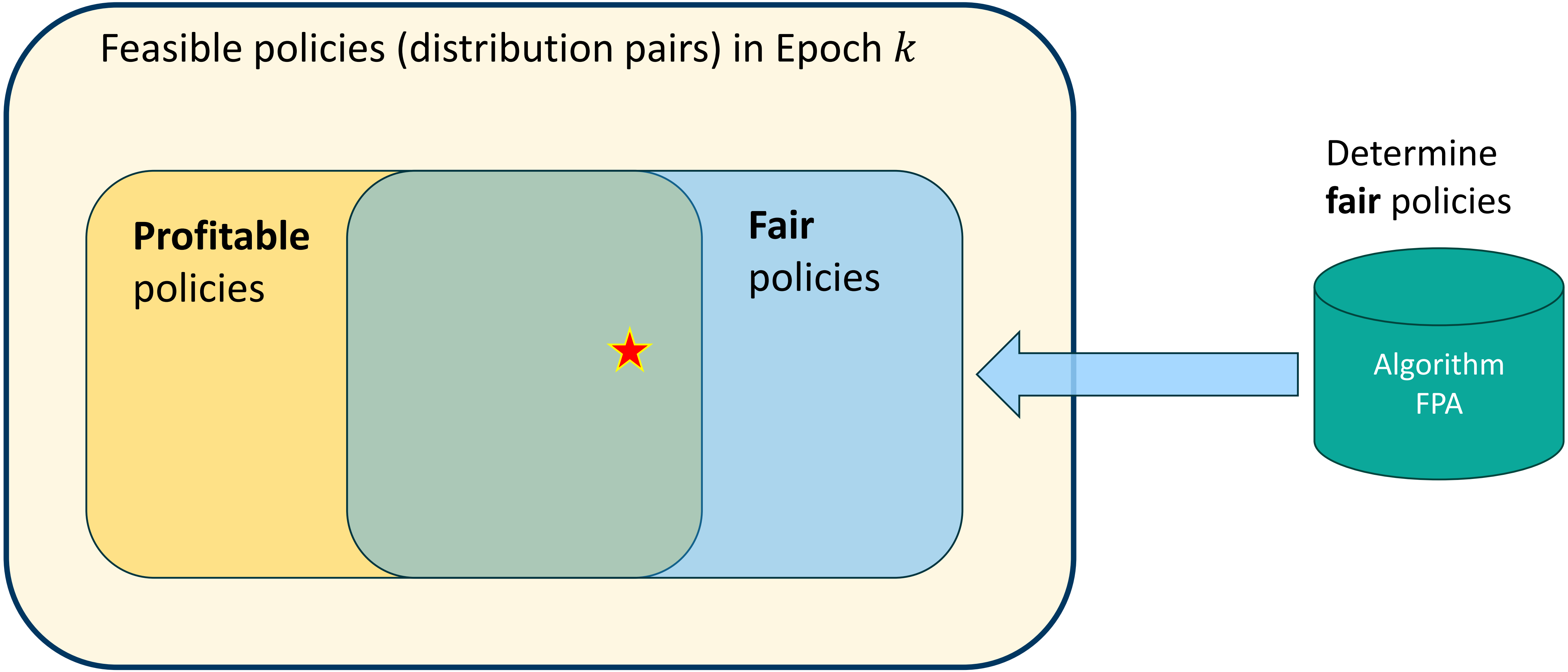

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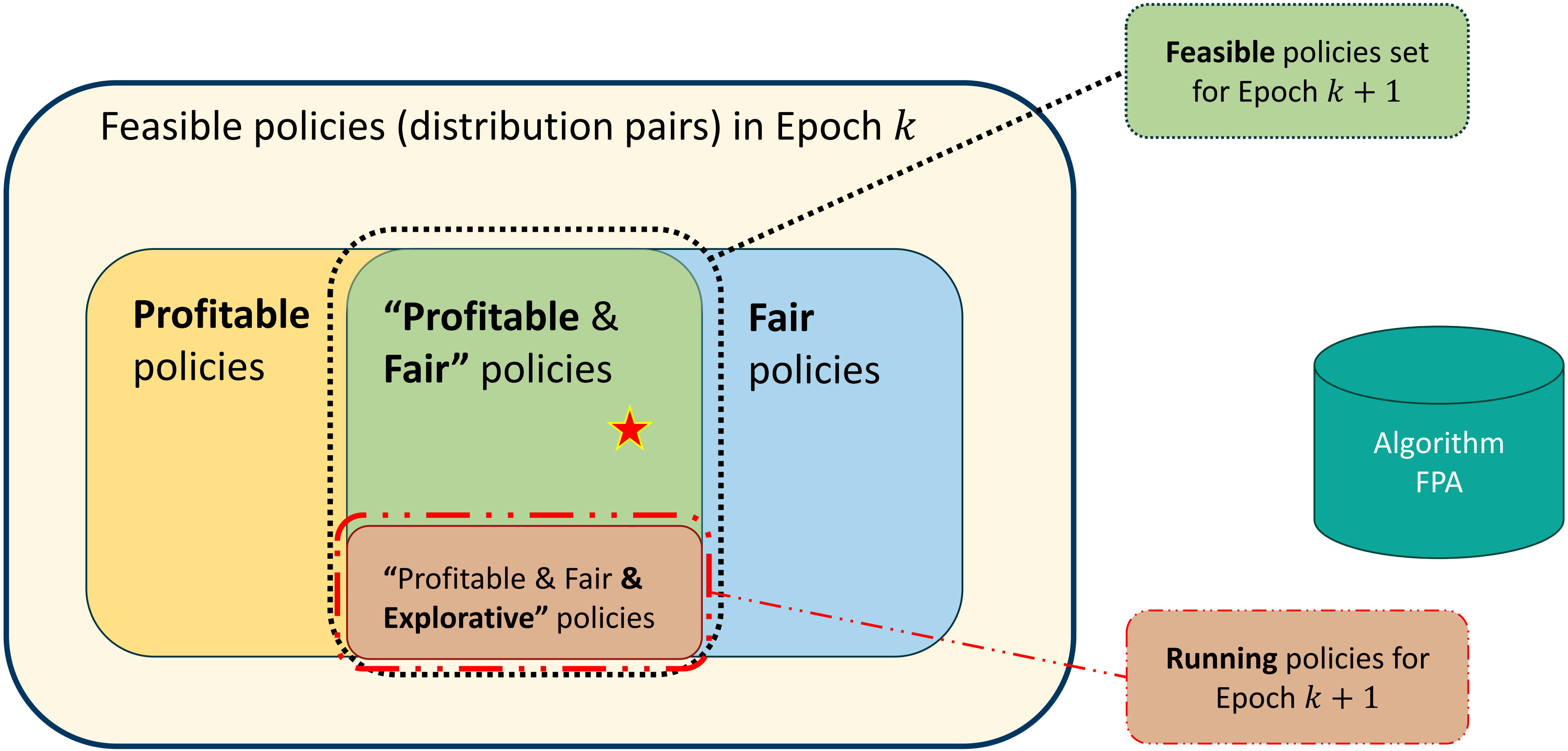

Optimal Regret, Optimal Unfairness, and Optimal Regret-Unfairness Tradeoffs

- Our FPA algorithm guarantees …
  - $\tilde{O}(\sqrt{T})$ regret
  - **0** procedural unfairness
  - $\tilde{O}(\sqrt{T})$ substantive unfairness

- To show the optimality, we also prove lower bounds of …
  - $\Omega(\sqrt{T})$ regret
  - Necessarily unfair: $\boldsymbol{O(\sqrt{T})}$ **regret** $\Rightarrow$ $\boldsymbol{\Omega(\sqrt{T})}$ **substantive unfairness**.

Conclusions and Potential Extensions

- Two fairness concerns:
  - Procedural and Substantive fairness
- Randomized prices might be doubly-fair and more profitable
- To solve the online fair pricing problem, we
  - Propose an FPA algorithm
  - Prove its regret and unfairness guarantees
  - Show the optimality of regret & unfairness & regret-unfairness tradeoffs.

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